**6. VECTOR CALCUSUS**

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| **DOT/SCALAR PRODUCT:** | **CORSS/ VECTOR PRODUCT:** |
| **DIVERGENCE OF FUNCTION:** |  |
| **CURL OF FUNCTION:** |  |

1. Gradient of a scalar function gives vector function.
2. Gradient of a scalar function is rate of change of function with respect to “x”, “y”, “z”.
3. From a scalar field, we can obtain vector filed by gradient.

**APPLICATION OF GRADIENT OF A SCALAR FUNCTION:**

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| 1. | The unit normal vector to the surface at point p is given by, |  |
| 2. | The directional derivative of surface at point p in the direction of is given by, |  |
| 3. | The maximum value of the directional derivative of surface at point p is given by, |  |

**SUMMARY OF DIVERGENCE OF FUNCTION:**

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| 1. | The divergence of the vector field given by, |  |
| 2. | From a vector field, we can obtain scalar filed by gradient. |  |
| 3. | The divergence of the gradient is Laplacian. |  |
| 4. | The divergence measures outflow minus in flow. | |
| 5. | is referred to solenoidal or divergence free, if | |

**SUMMARY OF CURL:**

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| 1. | is Curl free/ irrotational vector, if |  |
| 2. | Gradient field are irrotational. |  |
| 3. | Divergence of curl of a vector function is zero. |  |

**INTRODUCTION TO VECTOR INTEGRATION:**

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| Line Integration: | Line/Contour Integration: |
| Double Integration: | Closed Contour Integration: |
| Double Integration: | Surface Integration: |
| Triple Integration: | Closed Surface Integration: |

Area of a triangle formed by the tips of vectors

Vector triple product of three vectors

The parametric representation of the curve C is given by,

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1) In the aerodynamics and fluid mechanics is called circulation of around C where .

2) Work done by Force: is a force acting on a particle which moves a point to , along the line integral gives the total amount of work done by .

The value of the integral of a vector point function depends on the path C joining and (unless the vector function is irrotational.)

If the is a conservative field or an irrotational vector (i.e. ) in a region “R” then

1. The line integral is independent of path C joining and in region “R” and
2. around any closed curve C in region “R”.

**GREENS THEOREM:**

Let “R” be a closed bounded region in the XY-plane whose boundary C. Let be a vector function such that and are functions that are continuous and have continuous partial derivatives. Then

**SURFACE INTEGRAL:**

Consider a surface “S”. Divide the area “S” into “M” elements of area where p = 1, 2, 3, …, M. Choose any point coordinates are Let and be the positive unit normal vector to and . Form the sum . Where, is normal component of at . Now take the limit of this sum as M→∞. This limit is called surface integral of the normal component of over S and is denoted by .

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Method of evolution of surface integral:

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| If is the projection of S on XY-plane |  |
| If is the projection of S on YZ-plane |  |
| If is the projection of S on ZX-plane |  |

**STOKES THEOREM (LINE INTEGRAL TO SURFACE INTEGRAL):**

If is a differential vector function defined open surface S bounded by a simple closed curve C,

Where, is the outward unit normal vector to the surface S.

**GAUSS DIVERGENCE THEOREM (CLOSED SURFACE INTEGRAL TO VOLUME INTEGRAL):**

If is a differential vector function defined open surface S enclosing volume V,